BEST SELLER EBOOK

GAUAHTI UNIVERSITY

BUSINESS MATHEMATICS

SELF STUDY NOTES 2024

EBook Contents

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BUSINESS MATHEMATICS NOTES
UNIT-1 MATRICES AND DETERMINANTS

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1. Define determinant. [GU B.COM]

Ans: An arrangement of numbers along horizontal lines (called rows) and vertical lines called columns enclosed by two vertical lines is said to be a determinant. There are equal number of rows and columns in a determinant. It is denoted by D or Δ.

2. What is order of determinant?

Ans: The number of rows (or columns) in a determinant is called the order of the determinant.

Ex: $\begin{vmatrix} 1 & 2 \\ a & 4 \end{vmatrix} \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_1 & c_1 \\ a_3 & b_3 & b_3 \end{vmatrix}$ are determinant of order.

2 and 3 respectively.

Expansion of determinants:
(a) Second order determinants

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

$$\text{Ex: } \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = 2.5 - 3.4 = 10 - 12 = -2$$

Third order determinant:

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$
$$= a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)$$

Ex: $\begin{vmatrix} 1 & 2 & -3 \\ -2 & 3 & 4 \\ 2 & -1 & 5 \end{vmatrix} = 1 \begin{vmatrix} 3 & 4 \\ -1 & 5 \end{vmatrix} - 2 \begin{vmatrix} -2 & 4 \\ 2 & 5 \end{vmatrix} + (-3) \begin{vmatrix} -2 & 3 \\ 2 & -1 \end{vmatrix}$

Q1. Find the co-factor of the element 5 in the determinant

[GU B.COM]

$$\begin{vmatrix} 4 & 9 & 2 \\ 3 & 5 & 7 \\ 8 & 1 & 6 \end{vmatrix}$$

Soln: Co-factor of 5 $= (1)^{2+2} \begin{vmatrix} 4 & 2 \\ 8 & 6 \end{vmatrix}$

$$= (-1)^4(24 - 16)$$
$$= 8$$

Q2. Evaluate the determinant:

(a) $\begin{vmatrix} 0 & 2 & 3 \\ 1 & 4 & 7 \\ 2 & 0 & 4 \end{vmatrix}$

(b) $\begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \\ 0 & 1 & 2 \end{vmatrix}$

[GU B.COM]

Soln : (a) $\Delta = \begin{vmatrix} 0 & 2 & 3 \\ 1 & 4 & 7 \\ 2 & 0 & 4 \end{vmatrix}$

$$= 0\begin{vmatrix} 4 & 7 \\ 0 & 4 \end{vmatrix} - 2\begin{vmatrix} 1 & 7 \\ 2 & 4 \end{vmatrix} + 3\begin{vmatrix} 1 & 4 \\ 2 & 0 \end{vmatrix}$$

$$= 0 - 2 \cdot (4 - 14) + 3(0 - 8)$$
$$= -2(-10) + 3(-8)$$
$$= 20 - 24$$
$$= -4$$

(a) $\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \\ 0 & 1 & 2 \end{vmatrix}$

$$= 1\begin{vmatrix}1 & 2\\1 & 2\end{vmatrix} - 2\begin{vmatrix}3 & 2\\0 & 2\end{vmatrix} + 1\begin{vmatrix}3 & 1\\0 & 1\end{vmatrix}$$
$$= 1(2-2) - 2(6-0) + 1(3-0)$$
$$= 0 - 12 + 3$$
$$= -9$$

Q3. Solve the following: **[GU B.COM]**

(i) $\begin{vmatrix}x & 0 & 0\\2 & 1 & 3\\0 & 1 & 4\end{vmatrix} = 3$ (ii) $\begin{vmatrix}x^2 & x & 1\\0 & 2 & 1\\3 & 1 & 4\end{vmatrix} = 28$ (iii) $\begin{vmatrix}x & a & b\\a & x & b\\a & b & x\end{vmatrix} = 0$

Soln ^{n} : (i) $\begin{vmatrix}x & 0 & 0\\2 & 1 & 3\\0 & 1 & 4\end{vmatrix} = 3$

$$\Rightarrow x\begin{vmatrix}1 & 3\\1 & 4\end{vmatrix} - 0\begin{vmatrix}2 & 3\\0 & 4\end{vmatrix} + 0\begin{vmatrix}2 & 1\\0 & 1\end{vmatrix} = 3$$
$$\Rightarrow x(4-3) - 0 + 0 = 3$$
$$\Rightarrow x \cdot 1 = 3$$
$$\Rightarrow x = 3 \text{ Ans.}$$

(ii) Soln. $\begin{vmatrix}x^2 & x & 1\\0 & 2 & 1\\3 & 1 & 4\end{vmatrix} = 28$

$$\Rightarrow x^2\begin{vmatrix}2 & 1\\1 & 4\end{vmatrix} - x\begin{vmatrix}0 & 1\\3 & 4\end{vmatrix} + 1\begin{vmatrix}0 & 2\\3 & 1\end{vmatrix} = 28$$
$$\Rightarrow x^2(8-1) - x(0-3) + 1(0-6) = 28$$
$$\Rightarrow x^2 \cdot 7 + 3x - 6 = 28$$
$$\Rightarrow 7x^2 + 3x - 6 - 28 = 0$$
$$\Rightarrow 7x^2 + 3x - 34 = 0$$
$$\Rightarrow 7x^2 + 17x - 14x - 34 = 0$$
$$\Rightarrow x(7x+17) - 2(7x+17) = 0$$
$$\Rightarrow (7x+17)(x-2) = 0$$

either $7x + 17 = 0$ or $x - 2 = 0$

$$\Rightarrow x = -17/7 , \Rightarrow x = 2$$

(iii) $\begin{vmatrix} x & a & b \\ a & x & b \\ a & b & x \end{vmatrix} = 0$

$$\Rightarrow x\begin{vmatrix} x & b \\ b & x \end{vmatrix} - a\begin{vmatrix} a & b \\ a & x \end{vmatrix} + b\begin{vmatrix} a & x \\ a & b \end{vmatrix} = 0$$
$$\Rightarrow x(x^2 - b^2) - a(ax - ab) + b(ab - ax) = 0$$
$$\Rightarrow x(x + b)(x - b) - a^2(x - b) - b \cdot a(x - b) = 0$$
$$\Rightarrow (x - b)\{x(x + b) - a^2 - ab\} = 0$$
$$\Rightarrow (x - b)\{x^2 + xb - a(a + b)\} = 0$$
$$\Rightarrow (x - b)\{x^2 + (a + b - a)x - a(a + b)\} = 0$$
$$\Rightarrow (x - b)\{x^2 + (a + b)x - ax - a(a + b)\} = 0$$
$$\Rightarrow (x - b)[x\{x + (a + b)\} - a\{x + (a + b)\}] = 0$$
$$\Rightarrow (x - b)\{x + (a + b)\}(x - a) = 0$$

either

$x - b = 0,\ x - a = 0$ or $\quad x + (a + b) = 0$

$\Rightarrow x = b \ \Rightarrow x = a \qquad \Rightarrow x = -(a + b)$

Solving equations using determinants (Cramer's Rule):

Cramer's rule for three equations in three variables. Consider the system of three linear equation in three variables x, y, z.

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

Let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$

Then $\Delta_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$

$$\Delta_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$$

$$\Delta_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

$$\therefore x = \frac{\Delta x}{\Delta},\ y = \frac{\Delta y}{\Delta},\ z = \frac{\Delta z}{\Delta}$$

Q4. Solve by Cramer's Rule

(a) $3x - 5y = 7$
$4x + y = 17$

(b) $x + y + z = 3$ **[GU B.COM]**
$2x + 3y + 4z = 9$
$x + 2y - 4z = -1$

(c) $x + y + z = 3$

$2x - 3y + 5z = 4$

$z + 2y - 4z = -1$

Soln: (a) The given system of equation

$$3x - 5y = 7$$

$$4x + y = 17$$

$$\therefore \Delta = \begin{vmatrix} 3 & -5 \\ 4 & 1 \end{vmatrix} = 3 - (-20)$$

$$= 3 + 20 = 23 \neq 0$$

$\Delta_x = \begin{vmatrix} 7 & -5 \\ 17 & 1 \end{vmatrix} = 7 - (-5).17$

$= 7 + 85 = 92$

$$\Delta_y = \begin{vmatrix} 3 & 7 \\ 4 & 17 \end{vmatrix} = 51 - 28 = 23$$

$$\therefore x = \frac{\Delta x}{\Delta} = \frac{92}{23} = 4$$

and $\therefore y = \frac{\Delta y}{\Delta} = \frac{23}{23} = 1 \quad \therefore x = 4, y = 1$

Q1. Define function.

Ans: A function is a process or a relation that associates each element x of a set X the domain of the function, to a single element y of another set Y (possibly the same set) the codomain of the function. It is generally denoted by letters as f, g, h. I if y is a function of x then y is denoted by the symbol $f(x)$ i.e. $y = f(x)$. Here x is called the independent variable and y is called the dependent variable.

$$Ex: y = 2x + 1$$

Q2. If $f(x) = x^2 - 3x + 2$ find $f(3)$ and $f(0)$

Ans: Here $f(x) = x^2 - 3x + 2$

$$\begin{aligned} \therefore f(1) &= 1^2 - 3.1 + 2 \\ &= 1 - 3 + 2 \\ &= 0 \\ \therefore f(3) &= 3^2 - 3.3 + 2 \\ &= 9 - 9 + 2 \\ &= 2 \\ f(0) &= 0^2 - 3.0 + 2 \\ &= 2 \end{aligned}$$

Q3. If $f(x) = 2x^2 + x$ show that $\frac{f(a+b)-f(a)}{b} = 4a + 2b + 1$

Ans : Here, $f(x) = 2x^2 + x$

$$\begin{aligned} \therefore f(a+b) &= 2(a+b)^2 + (a+b) \\ &= 2(a^2 + 2ab + b^2) + a + b \\ &= 2a^2 + 4ab + 2b^2 + a + b \end{aligned}$$

and $f(a) = 2a^2 + a$

$$\therefore \text{ LHS } \frac{f(a+b) - f(a)}{b} = \frac{2a^2 + 4ab + 2b^2 + a + b - (2a^2 + a)}{b}$$

$$= \frac{2a^2 + 4ab + 2b^2 + a + b - 2a^2 - a}{b}$$

$$= \frac{4ab + 2b^2 + b}{b}$$

$$= \frac{b(2b^2 + 4a + 1)}{b}$$

$= 4a + 2b^2 + 1$

$=$ RHS showed.

Q4. If $f(x) = \frac{ax+b}{bx+a}$ prove that $f(x) \cdot f\left(\frac{1}{x}\right) = 1$

Ans : Given $f(x) = \frac{ax+b}{bx+a}$

$$\therefore f\left(\frac{1}{x}\right) = \frac{a \cdot \frac{1}{x} + b}{b \cdot \frac{1}{x} + a}$$

$$= \frac{\frac{a+bx}{x}}{\frac{b+ax}{x}} = \frac{a+bx}{x} \cdot \frac{x}{b+ax}$$

$$= \frac{(bx+a)}{(ax+b)}$$

LHS $\quad f(x)f\left(\frac{1}{x}\right)$

$$= \frac{ax+b}{bx+a} \cdot \frac{bx+a}{ax+b}$$

$= 1$

$=$ RHS proved.

Q5. If $h(x) = x^2 + 3x + 6$ and $g(y) = \frac{y}{1+y}$ find $g[h(2)]$

Ans : Given $h(x) = x^2 + 3x + 6$ and $g(y) = \frac{y}{1+y}$

$$\therefore g\{h(x)\} = \frac{h(x)}{1+h(x)}$$

$$= \frac{x^2+3x+6}{1+x^2+3x+6}$$

$$= \frac{x^2+3x+6}{x^2+3x+7}$$

$$\therefore g\{h(x)\} = \frac{(2)^2+3.2+6}{(2)^2+3.2+7}$$

$$= \frac{4+6+6}{4+6+7}$$

$$= \frac{16}{17}$$

Q6. If the function $f(x)$ is defined by $f(x) = \frac{1-x}{1+x}$ show that,

$$f\left(\frac{1-x}{1+x}\right) = x$$

Ans: Given function $f(x) = \frac{1-x}{1+x}$

$$\therefore f\left(\frac{1-x}{1+x}\right) = \frac{1-\frac{1-x}{1+x}}{1+\frac{1-x}{1+x}}$$

$$= \frac{\frac{1+x-(1-x)}{1+x}}{\frac{1+x+1-x}{1+x}} = \frac{1+x-1+x}{1+x+1-x}$$

$$= \frac{2x}{2}$$

$$= x$$

$$\therefore f\left(\frac{1-x}{1+x}\right) = x \text{ showed.}$$

Q7. $\phi(x) = \log \frac{1-x}{1+x}$ show that $\phi(a) + \phi(b) = \phi\left(\frac{a+b}{1+ab}\right)$

Ans: Since $\phi(x) = \log \frac{1-x}{1+x}$

$$\therefore \phi(a) = \log \frac{1-a}{1+a} \text{ and } \therefore \phi(b) = \log \frac{1-b}{1+b}$$

$$\text{LHS, } \phi(a) + \phi(b) = \log \frac{1-a}{1+a} + \log \frac{1-b}{1+b}$$

$$= \log \left(\frac{1-a}{1+a}\right)\left(\frac{1-b}{1+b}\right)$$

$$= \log \left(\frac{1-a-b+ab}{1+a+b+ab}\right)$$

$$\text{RHS } \phi\left(\frac{a+b}{1+ab}\right) = \log \frac{1-\frac{a+b}{1+ab}}{1+\left(\frac{a+b}{1+ab}\right)}$$

$$= \log \frac{1+ab-(a+b)}{1+ab}$$

$$= \log \frac{1+ab-a-b}{1+ab+a+b}$$

$$= \log \left(\frac{1-a-b+ab}{1+a+b+ab}\right)$$

$\therefore$ RHS = LHS showed.

Q8. If $f(x) = b\frac{x-a}{b-a} + a\frac{x-b}{a-b}$ prove that $f(a) + f(b) = f(a+b)$

Ans: Given $f(x) = b\frac{x-a}{b-a} + a\frac{x-b}{a-b}$

$$\therefore \; f(a) = b\frac{a-a}{b-a} + a \cdot \frac{a-b}{a-b}$$

$$= b \cdot 0 + a \cdot 1$$

$$= a$$

$$f(b) = b \cdot \frac{b-a}{b-a} + a\frac{b-b}{a-b}$$

$$= b \cdot 1 + 0$$

$$= b$$

Ans: Given function $f(x) = \frac{1-x}{1+x}$

$$\therefore f\left(\frac{1-x}{1+x}\right)=\frac{1-\frac{1-x}{1+x}}{1+\frac{1-x}{1+x}}$$

$$=\frac{\frac{1+x-(1-x)}{1+x}}{\frac{1+x+1-x}{1+x}}=\frac{1+x-1+x}{1+x+1-x}$$

$$=\frac{2x}{2}$$

$$=x$$

Q7. $\phi(x)=\log\frac{1-x}{1+x}$ show that $\phi(a)+\phi(b)=\phi\left(\frac{a+b}{1+ab}\right)$

Ans : Since $\phi(x)=\log\frac{1-x}{1+x}$

$$\therefore \phi(a)=\log\frac{1-a}{1+a} \text{ and } \therefore \phi(b)=\log\frac{1-b}{1+b}$$

LHS, $\phi(a)+\phi(b)=\log\frac{1-a}{1+a}+\log\frac{1-b}{1+b}$

$$=\log\left(\frac{1-a}{1+a}\right)\left(\frac{1-b}{1+b}\right)$$

$$=\log\left(\frac{1-a-b+ab}{1+a+b+ab}\right)$$

$$\text{RHS } \phi\left(\frac{a+b}{1+ab}\right)=\log\frac{1-\frac{a+b}{1+ab}}{1+\left(\frac{a+b}{1+ab}\right)}$$

$$=\log\frac{\frac{1+ab-(a+b)}{1+ab}}{\frac{1+ab+a+b}{1+ab}}$$

$$=\log\frac{1+ab-a-b}{1+ab+a+b}$$

$$=\log\left(\frac{1-a-b+ab}{1+a+b+ab}\right)$$

$\therefore$ RHS = LHS showed.

Q8. If $f(x) = b\frac{x-a}{b-a} + a\frac{x-b}{a-b}$ prove that $f(a) + f(b) = f(a+b)$

Ans : Given $f(x) = b\frac{x-a}{b-a} + a\frac{x-b}{a-b}$

$$\therefore f(a) = b\frac{a-a}{b-a} + a \cdot \frac{a-b}{a-b}$$
$$= b \cdot 0 + a \cdot 1$$
$$= a$$
$$f(b) = b \cdot \frac{b-a}{b-a} + a\frac{b-b}{a-b}$$
$$= b \cdot 1 + 0$$
$$= b$$

$$\therefore f(a) + f(b) = a + b$$
$$\therefore f(a+b) = b \cdot \frac{a+b-a}{b-a} + a \cdot \frac{a+b-b}{a-b}$$
$$= b \cdot \frac{b}{b-a} + a \cdot \frac{a}{a-b}$$
$$= \frac{b^2}{b-a} + \frac{a^2}{a-b}$$
$$= \frac{-b^2}{a-b} + \frac{a^2}{a-b}$$
$$= \frac{a^2-b^2}{a-b}$$
$$= \frac{(a+b)(a-b)}{(a-b)}$$
$$= a + b$$
$$\therefore f(a) + f(b) = f(a+b) = a + b$$

Q9. If $f(x) = \frac{x-1}{x+1}$ prove that $\frac{f(x)-f(y)}{1+f(x)f(y)} = \frac{x-y}{1+xy}$

Ans : Given $f(x) = \frac{x-1}{x+1}$

$$\frac{f(x)-f(y)}{1+f(x)f(y)}$$

LHS
$$=\frac{\frac{x-1}{x+1}-\frac{y-1}{y+1}}{1+\frac{x-y}{x+1}\cdot\frac{y-1}{y+1}}$$

$$=\frac{\frac{(x-1)(y+1)-(y-1)(x+1)}{(x+1)(y+1)}}{1+\frac{(x-1)(y-1)}{(x+1)(y+1)}}$$

$$=\frac{\frac{xy+x-y-1-(yx+y-x-1)}{(x+1)(y+1)}}{1+\frac{(x-1)(y+1)+(x-1)(y-1)}{(x+1)(y+1)}}$$

$$=\frac{xy+x-y-1-yx-y+x+1}{xy+x+y+1+xy-x-y+1}$$

$$=\frac{2(x-y)}{2(xy+1)}$$

$$=\frac{x-y}{xy+1}$$

= RHS proved.

Q10. If $f(x)=\frac{3x+2}{3x-2}$. prove that $\frac{f(x)+1}{f(x)-1}=\frac{3x}{2}$

Ans: Given $f(x)=\frac{3x+2}{3x-2}$

LHS $\frac{f(x)+1}{f(x)-1}$

$$=\frac{\frac{3x+2}{3x-2}+1}{\frac{3x+2}{3x-2}-1}$$

$$= \frac{\frac{3x + 2 + 3x - 2}{(3x - 2)}}{\frac{3x + 2 - (3x - 2)}{(3x - 2)}}$$

$$= \frac{6x}{3x + 2 - 3x + 2}$$

$$= \frac{6x}{4}$$

$$= \frac{3x}{2}$$

$=$ RHS proved.

Q11. The total cost function $c(x)$ of producing x items is given by

$$c(x) = 1000 + 5x \text{ when } 0 \leq x \leq 500$$
$$2000 + 4x \text{ when } 500 < x \leq 2000$$

Ans: Given $c(x) = 1000 + 5x\ 0 \leq x \leq 500$

For $x = 430$ $c(x) = 1000 + 5 \times 430$

$= 1000 + 2150$

$= 3150$

Again $c(x) = 2000 + 4x$ for $x = 1200$

$$c(1200) = 2000 + \times 1200$$
$$= 2000 + 4800$$
$$= 6800$$